MSC INTERNAL NOTE MSC-CF-R-68-18

DIGITAL ANALYSIS OF LM TERMINAL PHASE BACKUP CHARTS

Prepared by: Larry B McWhorter

Larry B. McWhorter
Orbital Procedures Section

Authorized for Distribution

W J North

Warren J. North
Chief, Flight Crew Support Division

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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PHASE BACKUP CHARTS

Approved by: Milton C. Contella

Milton C. Contella
Chief, Orbital Procedures Section

Paul C. Kramer

Paul C. Kramer
Chief, Flight Procedures Branch

J. W. Bilodeau

James W. Bilodeau
Assistant Chief for Crew Integration

W. J. North

Warren J. North
Chief, Flight Crew Support Division

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TABLE OF CONTENTS

1.0 SUMMARY

A digital analysis of the LM terminal phase rendezvous charts was performed to verify their in-plane accuracy and performance in the presence of system and application errors. This analysis indicated that the theoretical performance was about 1.05 times the two impulse theoretical minimum using the constraints of 2120 second transfer time and elevation of the target line-of-sight above the local horizontal (θ_{LOS}) of 26.8° at TPI. With system errors and application errors included the translation fuel required was about 1.4 times theoretical minimum. With no line-of-sight control or braking these errors resulted in a mean-miss distance of approximately 1235 feet.

2.0 INTRODUCTION

The digital analysis of the terminal phase backup charts was performed to show the ability of the backup rendezvous charts to compute the Terminal Phase Initiation (TPI) and Mid-course (M/C) maneuvers in the presence of system and application errors. A total of 120 Monte Carlo runs were made in this analysis using the nominal and five sets of dispersed initial conditions (IC's). This report summarizes the results of this analysis.

3.0 DISCUSSION

3.1 Study Rules

The following ground rules were used in this study.

(1) Both vehicles were in the same plane.

(2) The target orbit was circular at approximately 180 nautical miles.

(3) All burns were made along and normal to the line-of-sight.

(4) Two mid-course corrections were made.

(5) Impulsive thrust was assumed for TPI and M/C maneuvers.

(6) Finite thrust was used for braking and LOS control.

(7) Time of transfer was 2120 seconds ($\approx 140^{\circ}$ wt).

(8) Elevation of the target line-of-sight at TPI was 26.8 degrees.

3.2 Cases Investigated

The cases investigated included the nominal 15 nautical mile ΔH as well as five cases with dispersed state vectors at TPI. The conditions at TPI ($\theta_{los} = 26.8^{\circ}$) for each case are shown in the table below.

Case	Altitude (ft)	Altitude Rate (ft/sec)	Velocity (ft/sec)	Range (ft)	Range Rate (ft/sec)
Nominal	1002610	+0.1	25336.5	200409.6	-140.12
I	994742	-3.5	25342.0	218010.8	-151.70
II	997014	+1.9	25338.8	212924.3	-148.82
III	999191	+1.2	25340.3	208065.7	-147.66
IV	1009964	-1.3	25331.5	184730.6	-127.66
V	999685	+1.4	25336.3	207091.9	-143.61

3.3 Charts and Procedures

The rendezvous charts modeled in this study were preliminary LM-3 terminal phase rendezvous charts. The data taking sequence simulated in this study was the same for the TPI and the M/C corrections. The sequence

required that data be taken at two points for each maneuver. At the first point (A) the relative elevation angle was taken, and at the second point (B) the relative elevation angle, range, and range rate were taken. This data was then entered into the chart logic to obtain the necessary correction with the ΔV applied subsequently at the appropriate time.

After the last M/C maneuver, finite braking and LOS control were executed based on a braking schedule with the line-of-sight angular rate being kept within a deadband of $\pm$ .1 milliradian per second after the range was below 15,000 feet.

An example of the terminal phase rendezvous charts modeled in this study is included as Appendix A.

3.4 Digital Program

In order to perform this analysis, functions describing the backup rendezvous charts were programmed into a digital routine which integrated the equations of motion of the two bodies about a spherical planet. These functions simulated the procedures of using the backup rendezvous charts by the use of logic which incorporated elevation angle, range, and range rate at the times called for by the backup sequence of data taking. System errors were then applied to the data before being used to calculate the TPI and M/C maneuvers. An error in velocity application was also included.

The two quantities on which the backup charts are based, $\Delta\theta_{NOM}$ and $\left[\frac{\dot{R}}{R} 1000\right]_{NOM}$, are supplied to the routine in the form of table look ups. Both of these quantities are stored as a function of the second elevation

angle measurement. These tables were generated by starting the spacecraft and target at rendezvous and applying a constant velocity increment in various directions to obtain trajectories with different approach angles.

The equations used for TPI are as follows:

A. $$\Delta V_{(FORWARD)} = \left[\left(\frac{\dot{R}_{TPI}}{R_{TPI}}\right) R_B - \dot{R}_B\right] K$$

1. $\Delta V_{(FORWARD)}$ is the velocity increment needed along the line-of-sight.
2. R_{TPI} is the nominal range rate after an impulsive TPI velocity increment has been applied.
3. R_{TPI} is the nominal range at TPI.
4. R_B is the range observed at point B.
5. $\dot{R}_B$ is the range rate observed at point B.
6. K is an emperically derived constant which is a function of the nominal velocity increment of TPI.

B. $$\Delta V_{(UP/DOWN} = \left[\frac{R_{TPI}}{R_{B_{NOM}}}\right] \left[R_B \left(\frac{\Delta\theta_A - \Delta\theta_{NOM}}{\Delta T}\right)\right]$$

1. $\Delta V_{(UP/DOWN)}$ is the velocity increment needed normal to the line-of-sight.
2. $R_{B_{NOM}}$ is the nominal range at point B.
3. ΔT is the change in time from point A to point B.
4. $\Delta\theta_{NOM}$ is the nominal change in the relative elevation angle between measurements.
5. $\Delta\theta_A$ is the observed change in the relative elevation angle between observations.

The equations used for the M/C corrections are as follows:

A. $\Delta V_{(FORWARD)} = \left(\frac{\dot{R}}{R}\,1000\right)_{NOM}\frac{R_M}{1000} - \dot{R}_M$

1. $\left(\frac{\dot{R}}{R}\,1000\right)$ is the nominal value of $\frac{\dot{R}}{R}\,1000$ required.

2. $\dot{R}_M$ is the range rate observed.

3. R_M is the range observed.

B. $\Delta V_{(UP/DOWN)} = R_M\left(\frac{\Delta\theta_A - \Delta\theta_{NOM}}{\Delta T}\right)$

3.5 Errors

Two types of errors were used in this simulation, (1) random errors and (2) constant errors. The constant errors and their values were as follows:

1.	Initial IMU pitch bias	$\pm$ 0.5 degrees
2.	IMU drift	$\pm$ 0.5 degrees/hour
3.	Time delay in reading consecutive quantities	10 seconds

The random errors and their standard deviations were as follows:

1.	Per axis application of burns	0.1 ft/sec
2.	Percent error in radar range	.25 percent
3.	Percent error in radar range rate	.25 percent
4.	Elevation angle	0.1 degree
5.	Time measurements	1.0 second
6.	Errors in reading tapemeter	
a.	Range rate	1.0 ft/sec
b.	Range on outer scale	3000 feet
c.	Range on middle scale	50 feet

If a correction in an axis was computed to be less than 1.0 ft/sec, the correction was not applied in that axis.

3.6 Trajectories

During this analysis, a total of 240 trajectory runs were made. The first 120 runs consisted of the TPI and M/C maneuvers omitting braking or line-of-sight control in order to establish the closest approach. These runs were repeated with braking and line-of-sight control included to obtain the fuel required by each run. Runs 1-60 were made with an initial IMU pitch bias of +0.5 degrees and a drift rate of +0.5 degrees per hour which caused TPI to occur early. Runs 61-120 were made with an initial IMU pitch bias of -0.5 degrees and a drift rate of -0.5 degrees per hour which caused TPI to occur late. All runs were initiated 15 minutes before the nominal time of TPI. Each of the 120 runs had different sets of random errors.

3.7 Braking Schedule

The braking schedule used in this simulation consisted of five gates and a lower limit on the range rate. The first gate was at 15,000 feet. At this point normally only line-of-sight control was executed because the allowed range rate was set at 80 feet per second. The second gate was at 6,000 feet with an allowed range rate of 40 feet per second. The nominal range rate of this range was 36 feet per second. The remaining gates were 20 feet per second at 3,000 feet, 10 feet per second at 1500 feet and 5 feet per second at 500 feet. The lower range rate limit consisted of a straight line connecting 20 feet per second at 15,000 feet with 0 feet per second at intercept. Both the upper and lower range rate limits are shown in Figure 1.

4.0 Results

4.1 Theoretical Performance

The theoretical performance of the charts was obtained by running each case without errors with zero deadband on the ΔV to be applied, both with and without braking. The results of these runs are shown on Table I. In Table I up, and aft are shown as positive and down and forward are shown as negative. The impulsive braking column gives the vectored ΔV required to null relative velocites at the point of closest approach. The ratio of the mean impulsive braking plus TPI and M/C ΔV to the mean theoretical minimum is 1.05.

4.2 Performance With Errors

4.2.1 Miss Distance

The magnitude of the miss distance without braking or line-of-sight control of each run is shown on Figures 2-4.

Figure 2 shows the distribution of the miss distance over a set of intervals. As can be seen from Figure 2, 30 percent of the runs have a miss distance of less than 500 feet, and less than 10 percent have a miss distance of greater than 2,500 feet. The mean miss distance of all 120 runs without braking or line-of-sight control was 1,234 feet.

Figures 3 and 4 show the miss distance of each run except for run number 37 which had a miss distance of 13,000 feet, three times as large as the miss distance of the next worse case.

The mean-miss distance for each case is shown below.

Case	Mean-Miss Distance (ft)
Nominal	976.4
I	1044.8
II	1207.3
III	1802.5
IV	977.6
V	1399.2

The mean-miss distance for case III is larger than the others because run number 37 was included. Without run 37, the mean for case III is 1109.5 feet. The mean-miss distance when TPI occurred early was 1724.7 feet compared to a mean-miss distance of 747.8 feet when TPI occurred late.

4.2.2 ΔV Used

For the 120 runs made, the mean total translational ΔV used was 97.8 feet per second. Of the 120 runs, 44 used a total ΔV between 92.5 and 102.5 feet per second, with only 4 runs requiring ΔV greater than 117.5 feet per second. Figures 5-7 show the distribution of ΔV. Figure 5 shows the distribution of ΔV used for TPI and M/C maneuvers, while Figure 6 shows the distribution of ΔV used for line-of-sight control and braking. Figure 7 shows the distribution of total ΔV used for all runs. The means are noted on each figure and are listed below.

Maneuver	Mean ΔV
TPI & M/C	53.99
Line-of-sight control	7.61
Braking	36.23
Braking and LOS	43.84
Total	97.83

The following table gives a comparison of theoretical minimum and mean actual ΔV used for various maneuvers of each case.

Case	Minimum	TPI and M/C	LOS	Braking	Total
Nominal	66.37	54.60	6.93	36.27	97.82
I	72.22	58.07	7.95	38.61	104.63
II	70.81	54.12	7.14	37.37	98.63
III	67.10	52.14	8.19	36.31	96.64
IV	62.43	49.17	7.60	32.40	89.17
V	69.52	55.82	7.87	36.40	100.19

Figures 8-13 show the ΔV required by each run for TPI and M/C braking and LOS control, and total ΔV required.

4.2.3 Transfer Time

The backup charts attempt to constrain the intercept to occur in 2120 seconds. This compares very closely with the mean transfer time from TPI to closest approach which was 2135 seconds. When braking and LOS control were executed, the mean time of transfer increased to 2319 seconds.

5.0 Conclusions

1. The backup rendezvous charts will satisfactorily compute the maneuvers necessary for terminal phase rendezvous.

2. The mean-miss distance could be reduced by adjusting the time of the second mid-course such that it would most likely occur within a range of 60,000 feet because of the additional accuracy of the tapemeter below 10 nautical miles.

3. The backup charts will satisfactorily control the time of transfer.

4. Transfers initiated earlier than nominal result in greater inaccuracies in the backup charts than those initiated late.

5. The mean in-plane translational ΔV required with systems and applications errors is 1.44 that of the mean theoretical minimum.

Table I

Case	Miss Distance (ft)	Transfer Time Without Braking (sec)	Transfer Time With Braking (sec)	Theoretical Minimum Lambert Solution (ft/sec)
Nominal	275.8	2130	2297	66.37
I	668.6	2140	2285	77.22
II	131.4	2130	2302	70.81
III	77.4	2130	2299	67.10
IV	529.0	2130	2292	62.43
V	163.5	2130	2301	69.52
Mean	307.6	2132	2296	68.08

Case	LOS ΔV	Finite Braking ΔV	Impulsive Braking ΔV	Total ΔV	$\frac{\text{Total } \Delta V}{\text{Theoretical Minimum}}$
Nominal	6.13	34.72	35.02	72.95	1.11
I	8.70	37.16	37.52	91.32	1.27
II	5.14	37.23	37.49	77.50	1.10
III	5.59	35.50	35.74	73.69	1.10
IV	8.14	32.05	32.49	74.42	1.20
V	5.05	36.60	36.85	77.45	1.08
Mean	6.46	35.54	35.85	77.89	1.14

	TPI		1st M/C ΔV		2nd M/C ΔV	
Case	F-A	U-D	F-A	U-D	F-A	U-D
Nominal	-29.90	-0.08	-0.87	0.33	-0.37	0.54
I	-31.78	3.43	-4.21	3.14	-1.08	1.82
II	-31.86	-1.83	-0.19	-0.77	-0.36	0.12
III	-29.69	1.09	0.45	-1.15	-0.06	-0.16
IV	-27.78	0.14	-2.61	1.17	-0.70	1.28
V	-31.87	-2.82	-0.13	-0.45	-0.36	0.20

FIGURE 1

BRAKING SCHEDULE

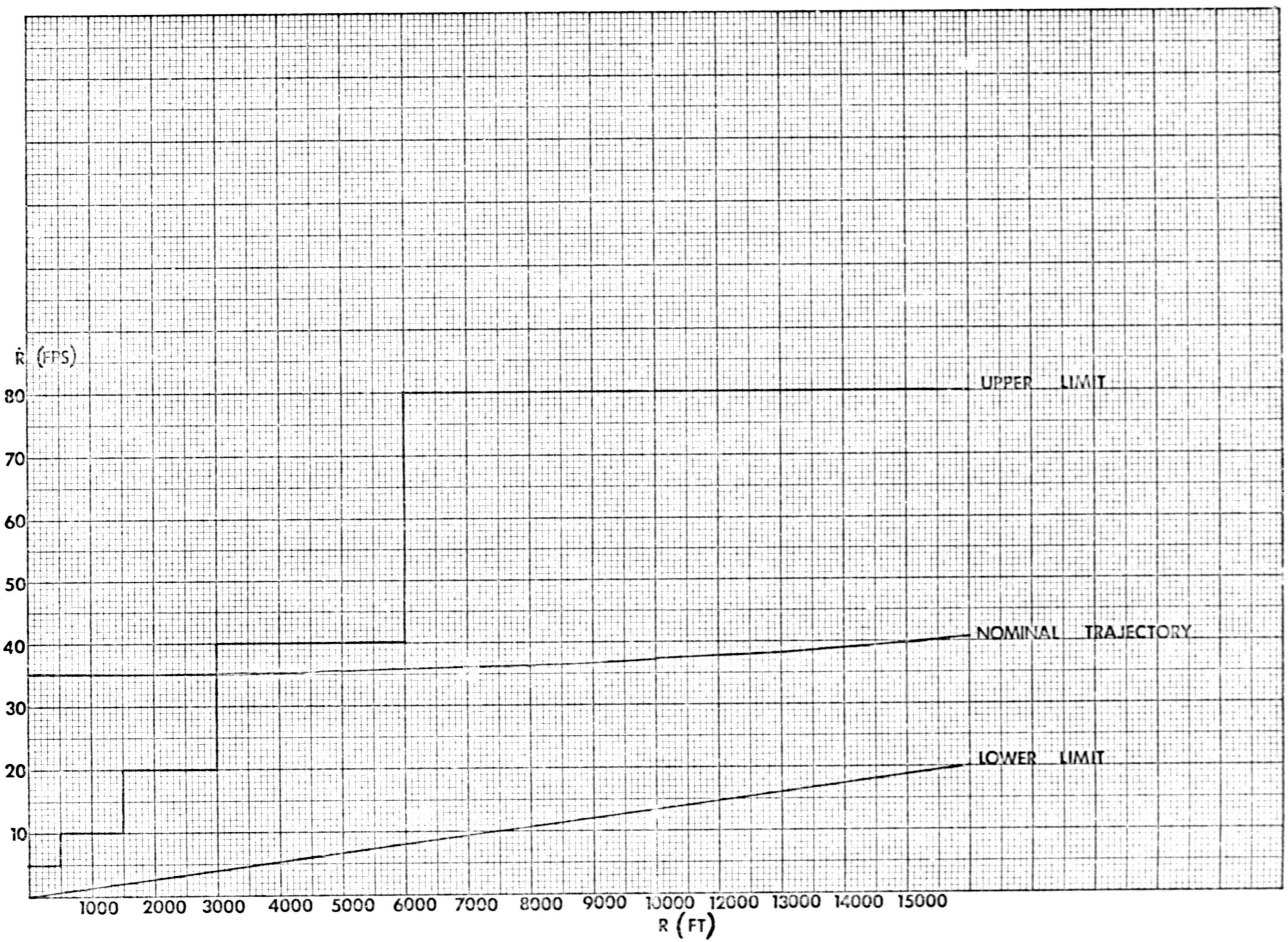
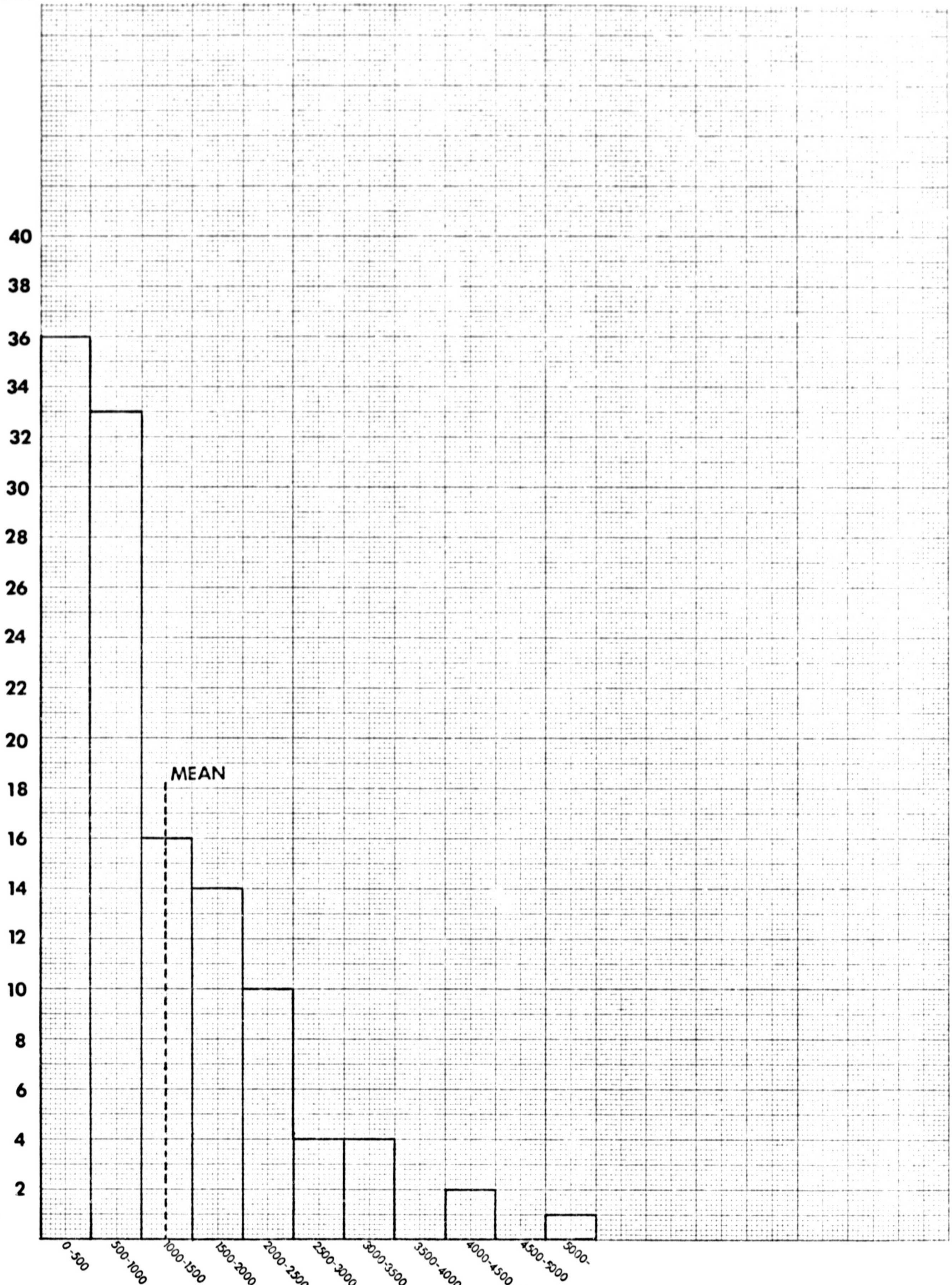
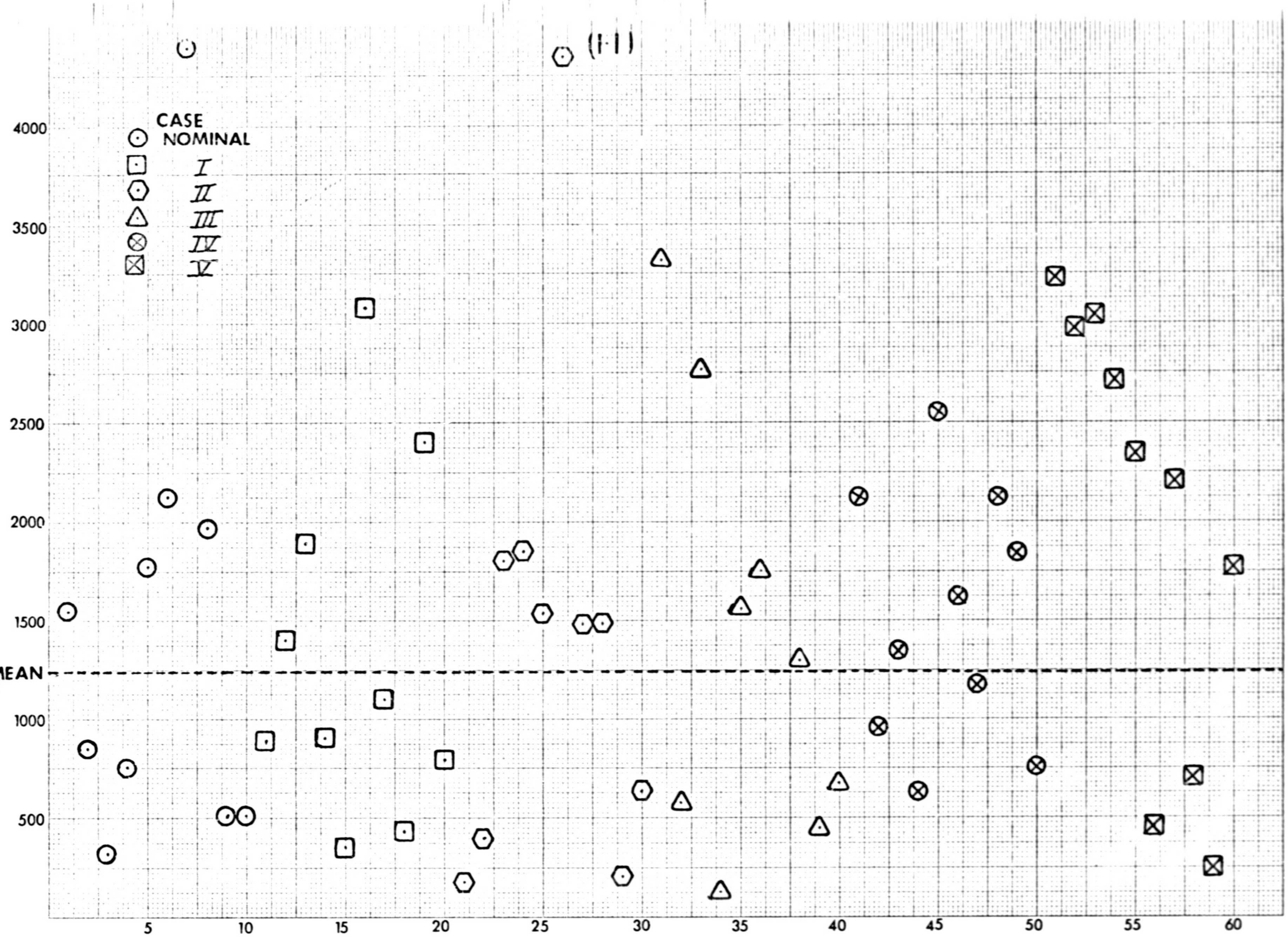


FIGURE 2

MISS DISTANCE
(FT)




(1·1)
CASE
NOMINAL
I
II
III
IV
V
4000
3500
3000
2500
2000
1500
MEAN
1000
500
5
10
15
20
25
30
35
40
45
50
55
60

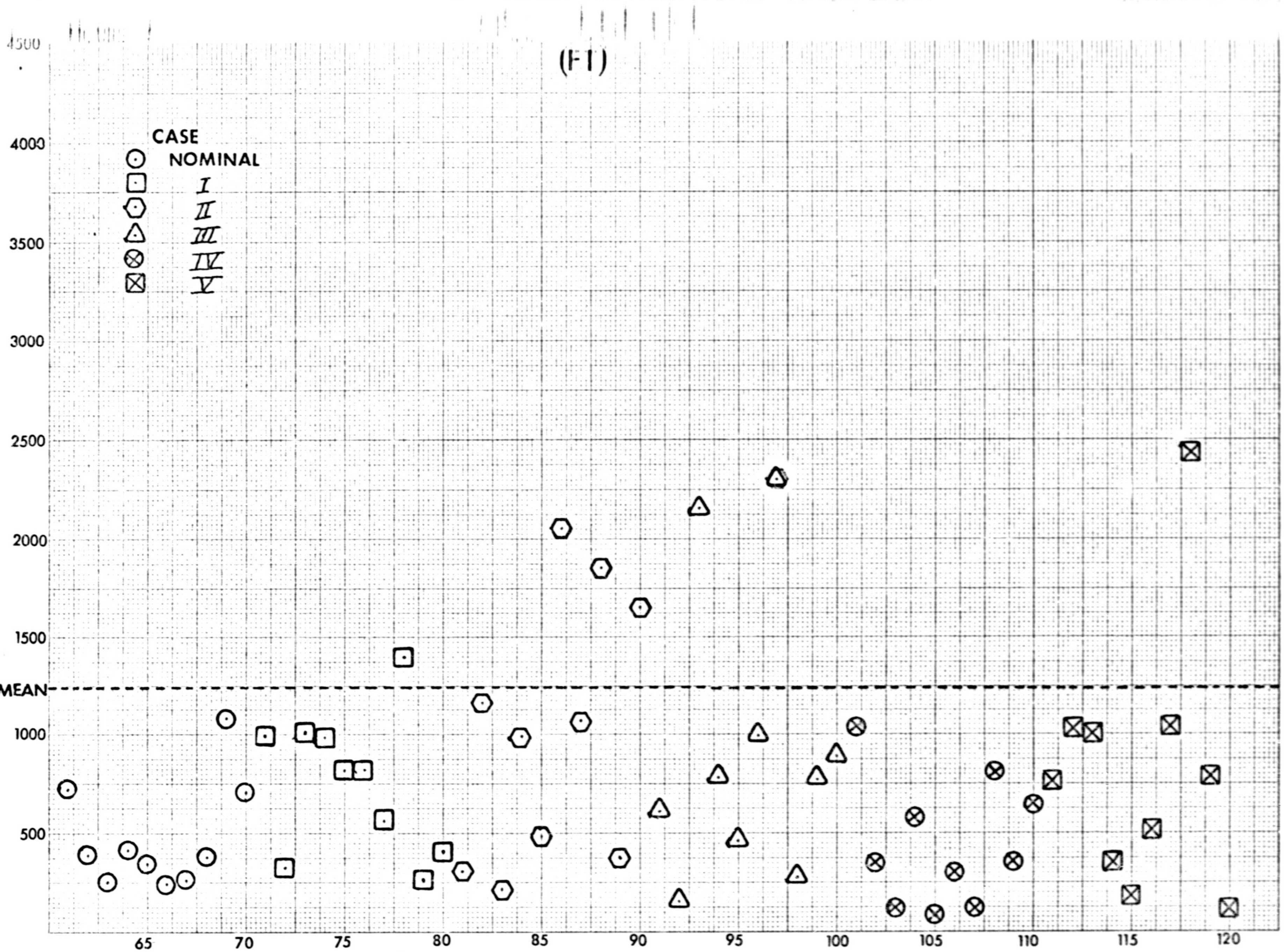

(F1)
CASE
NOMINAL
I
II
III
IV
V
4500
4000
3500
3000
2500
2000
1500
MEAN
1000
500
65
70
75
80
85
90
95
100
105
110
115
120

FIGURE 5

TPI AND M/C ΔV

(FPS)

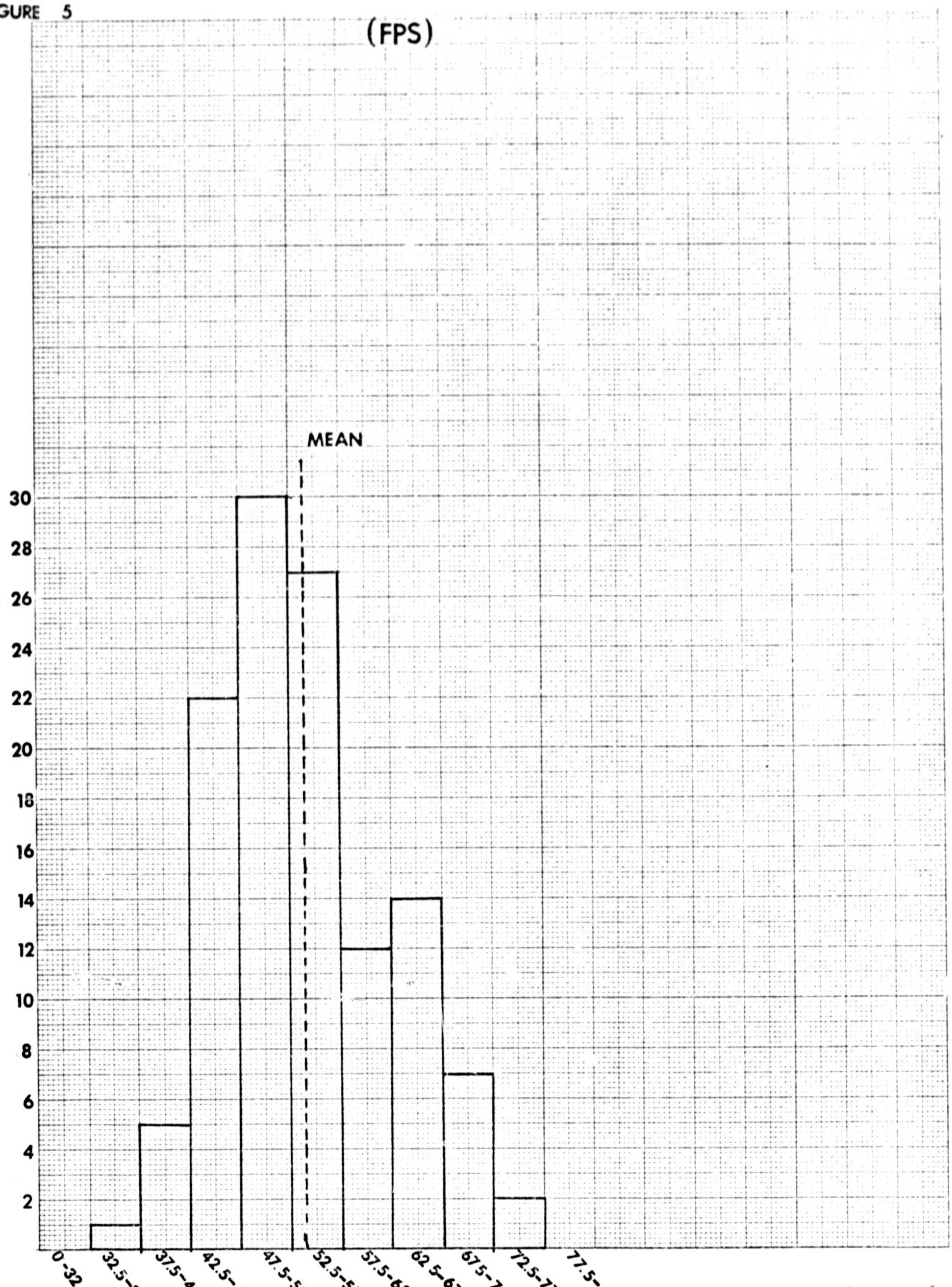


FIGURE 6

LOS AND BRAKING ΔV

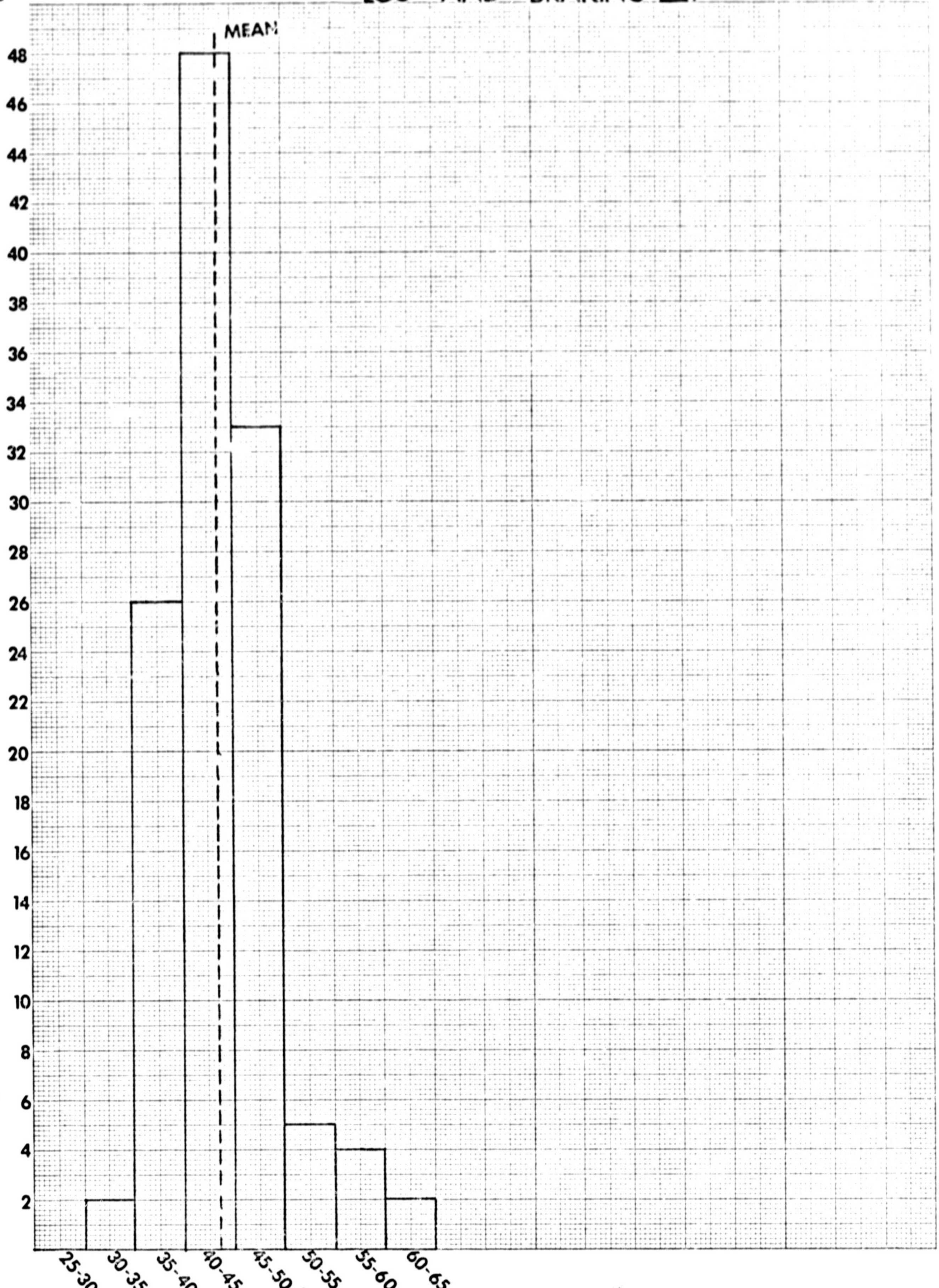


FIGURE 7

TOTAL ΔV

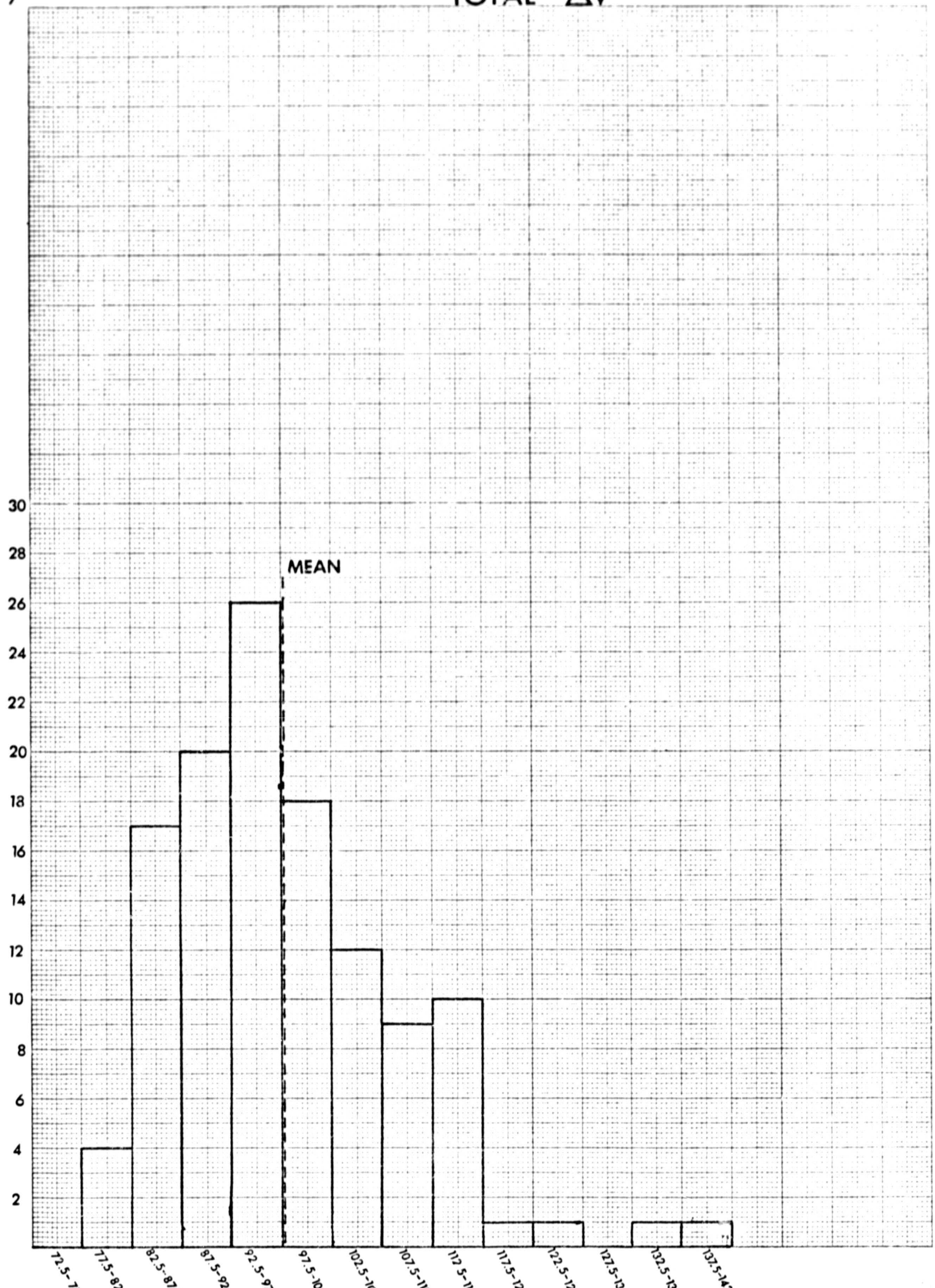


FIGURE 8

III A/B M/C AV

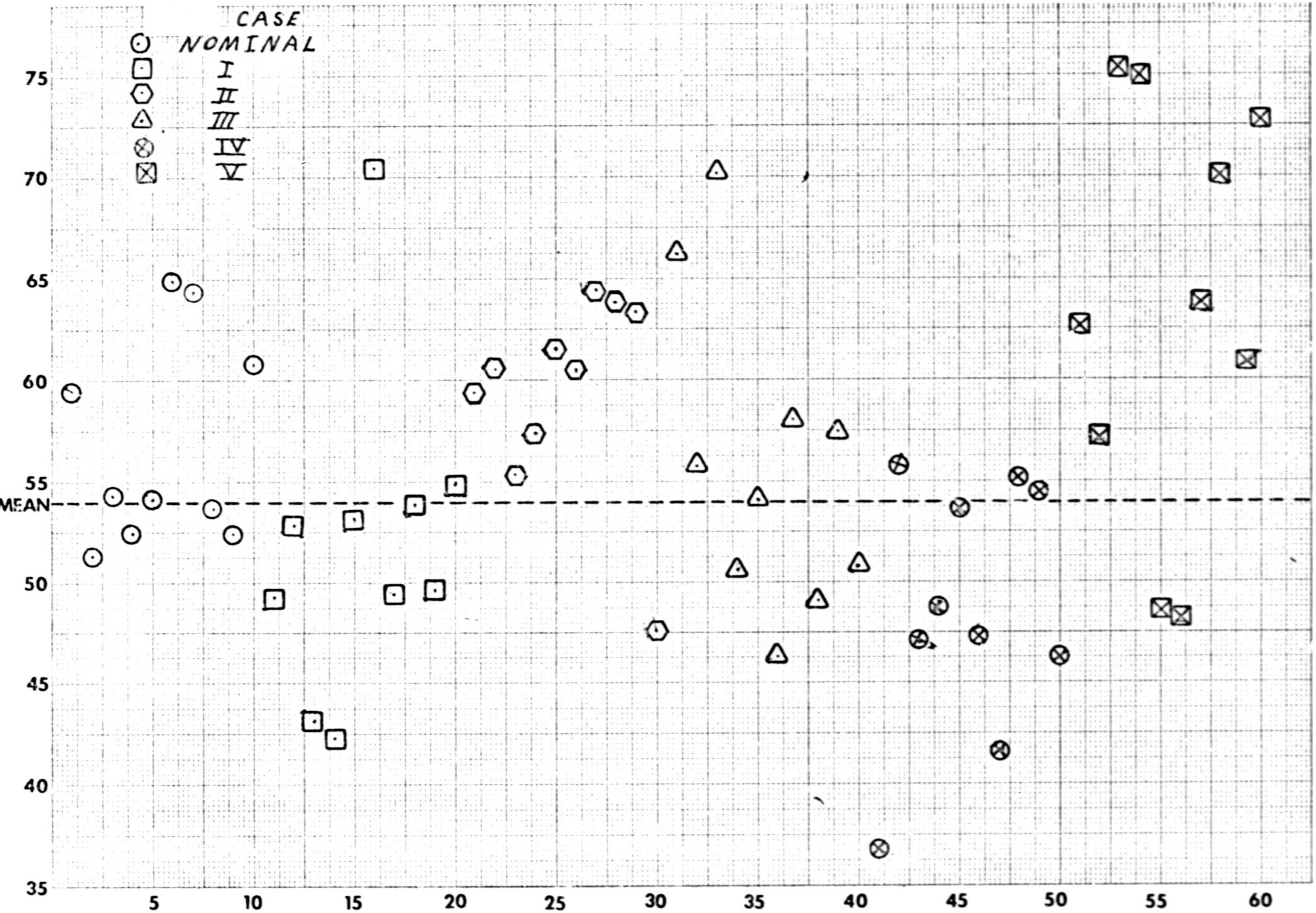
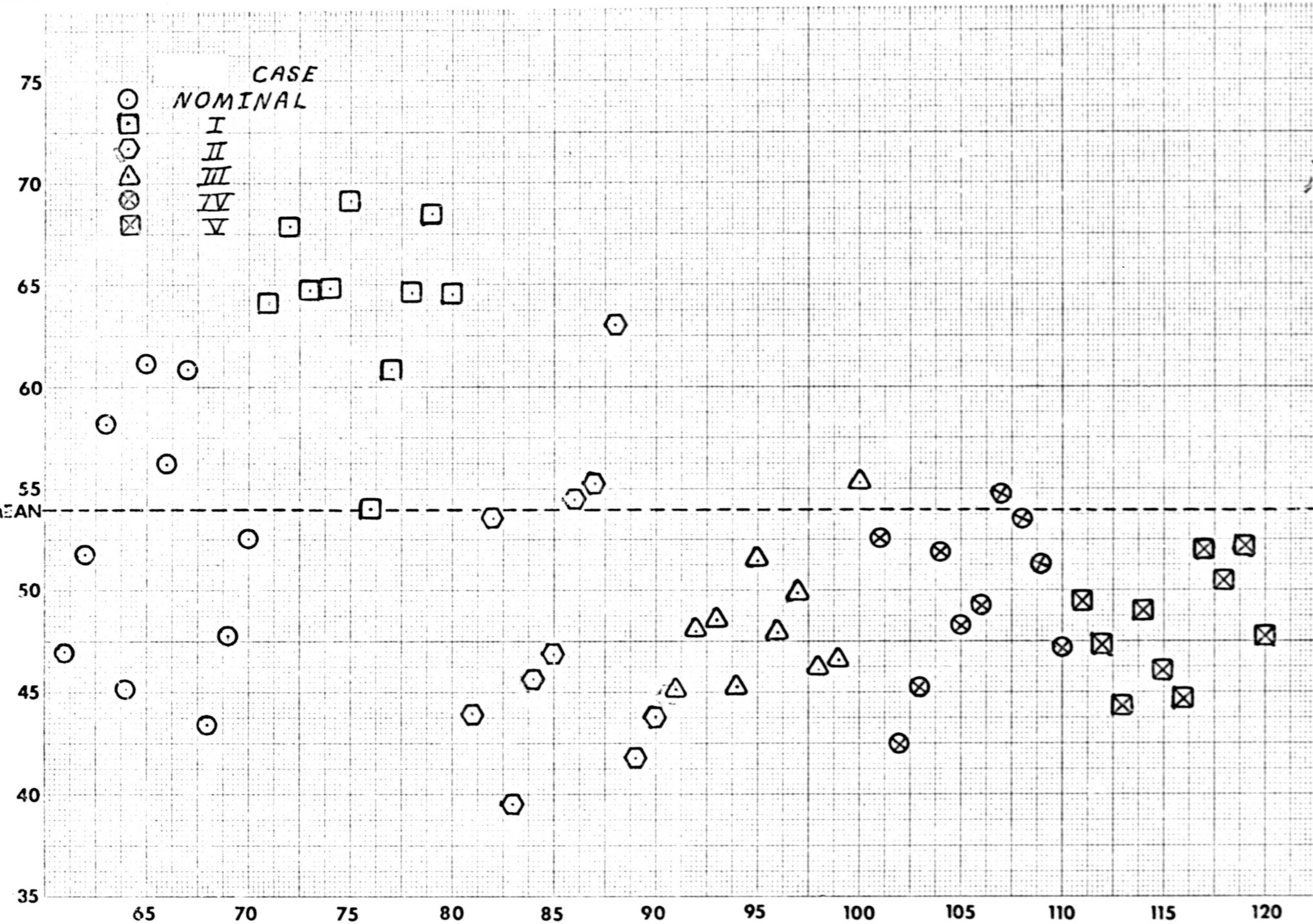


FIGURE 9



CASE
NOMINAL
I
II
III
IV
V
MEAN
75
70
65
60
55
50
45
40
35
65
70
75
80
85
90
95
100
105
110
115
120

FIGURE 10

BRAKING AND LOS ΔV

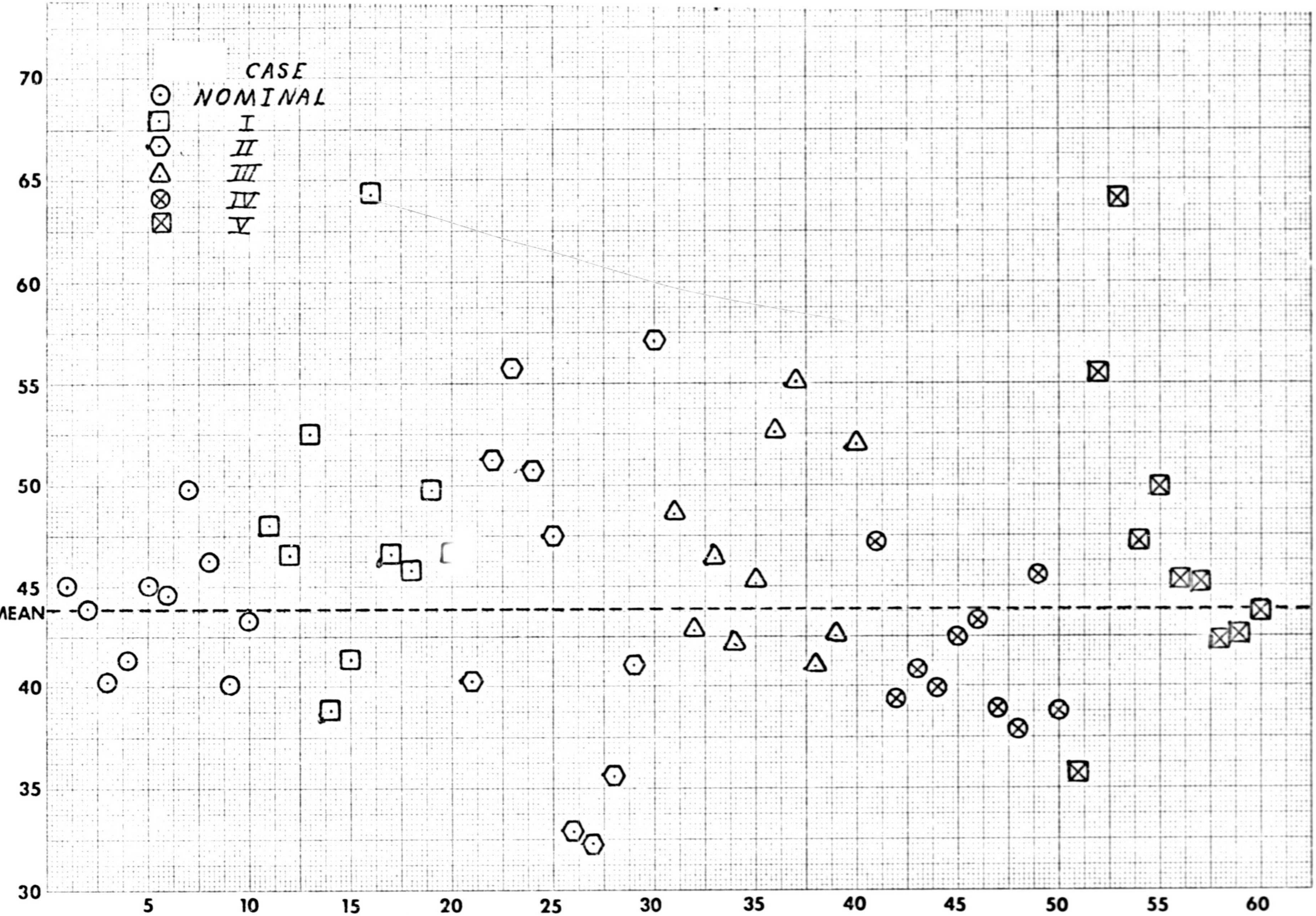


FIGURE 11

BRAKING AND LOS ΔV

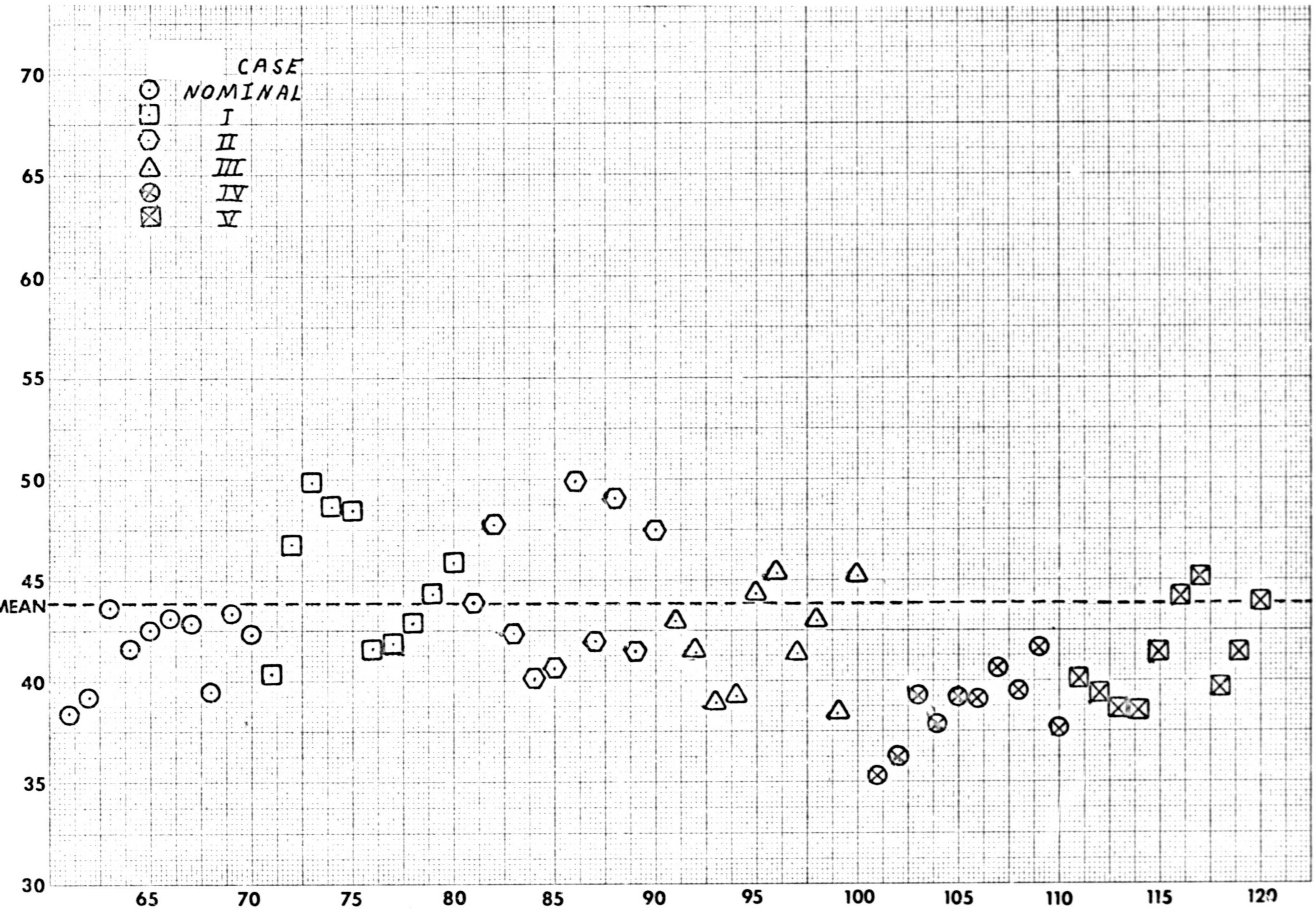
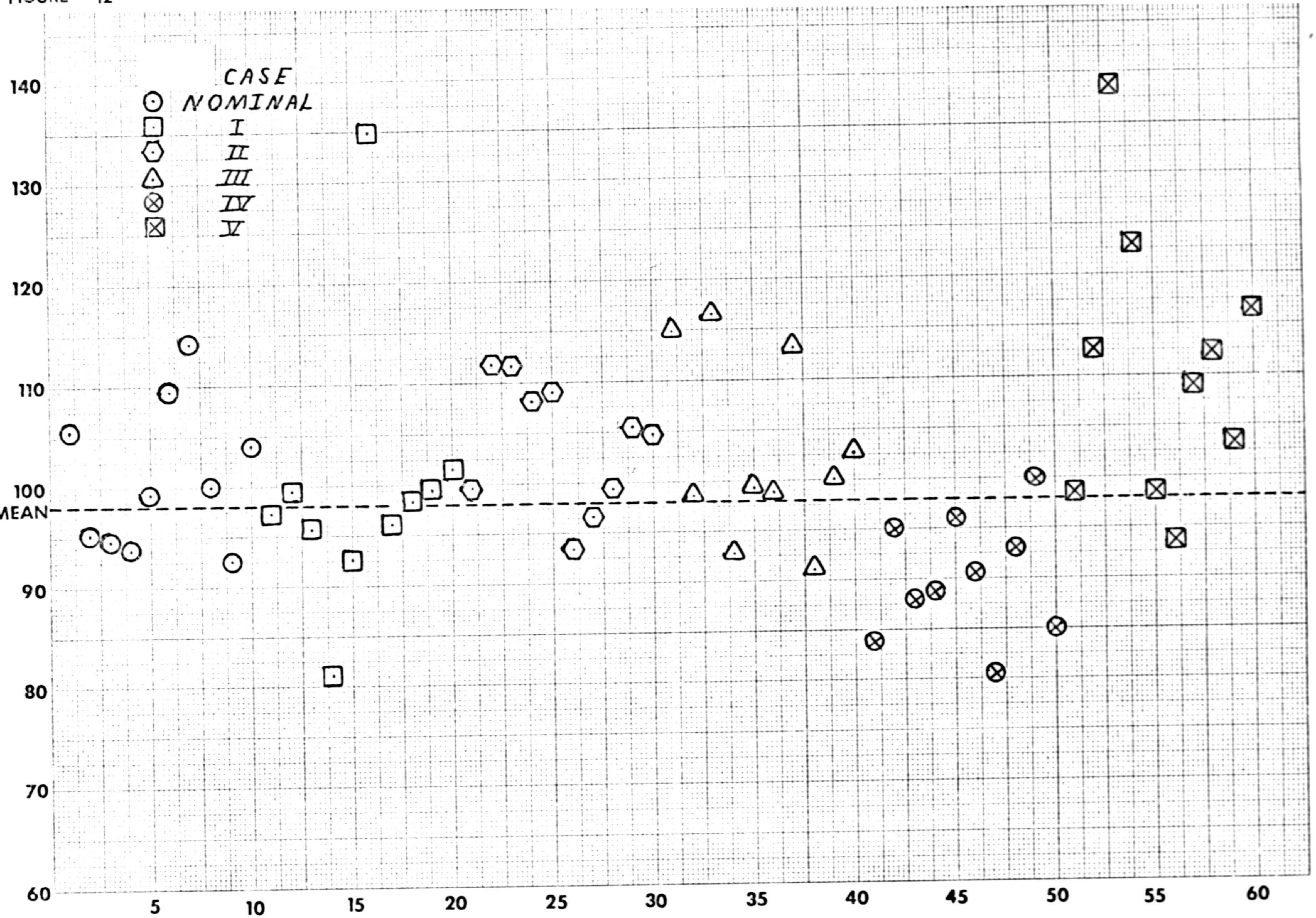
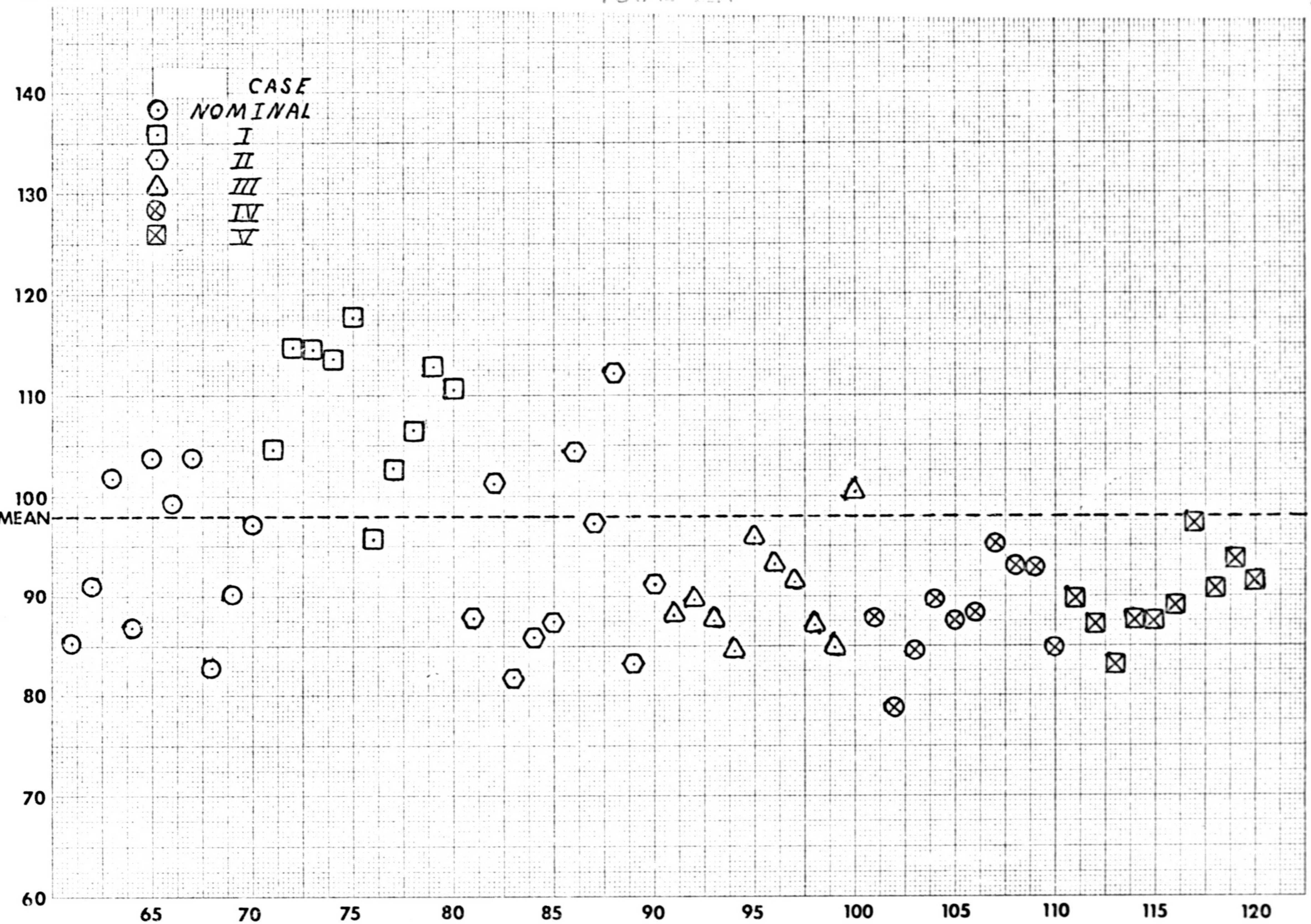


FIGURE 12

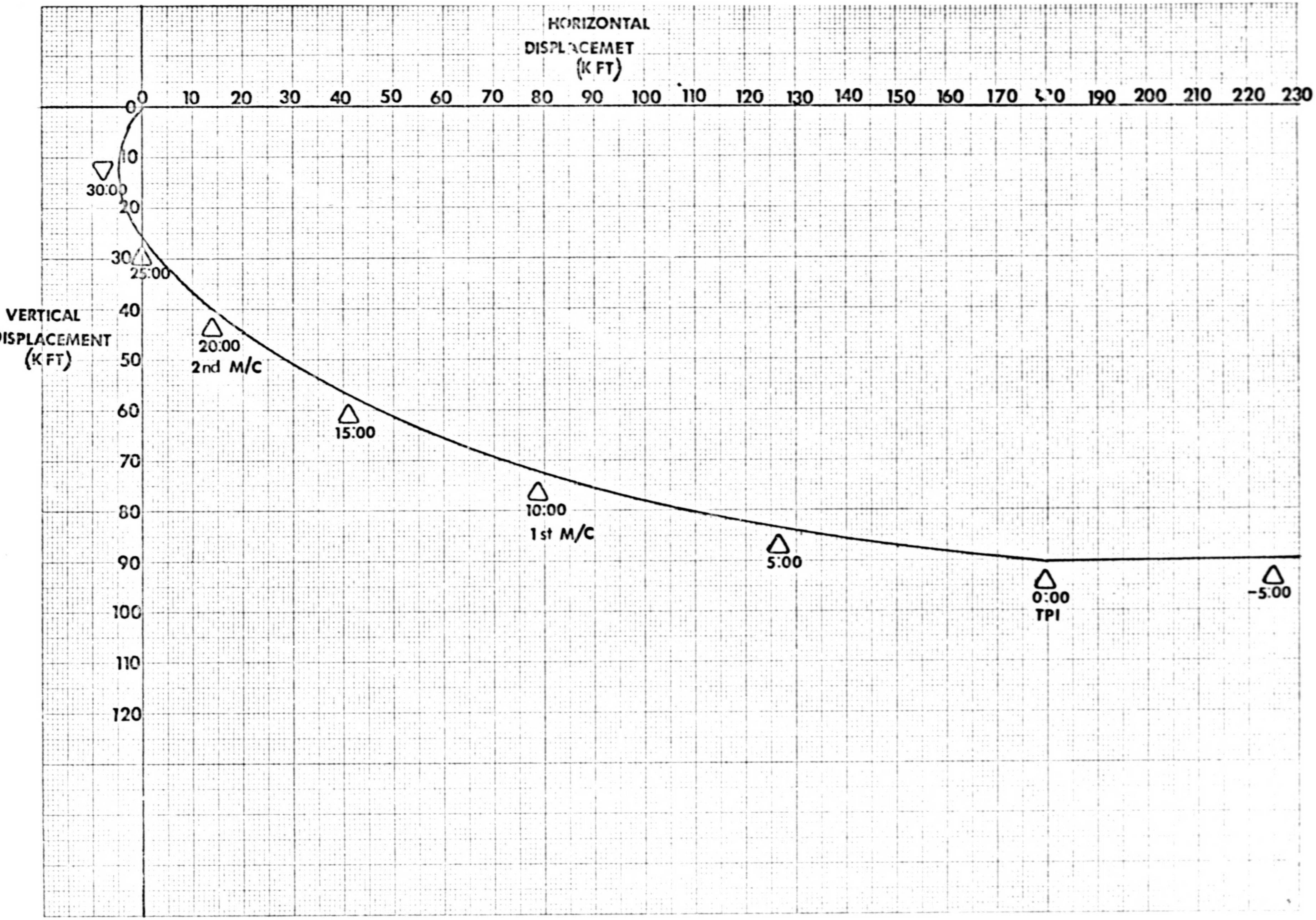
TOTAL ΔV





CASE
NOMINAL
I
II
III
IV
V
140
130
120
110
100
MEAN
90
80
70
60
65
70
75
80
85
90
95
100
105
110
115
120

LM RELATIVE MOTION PLOT



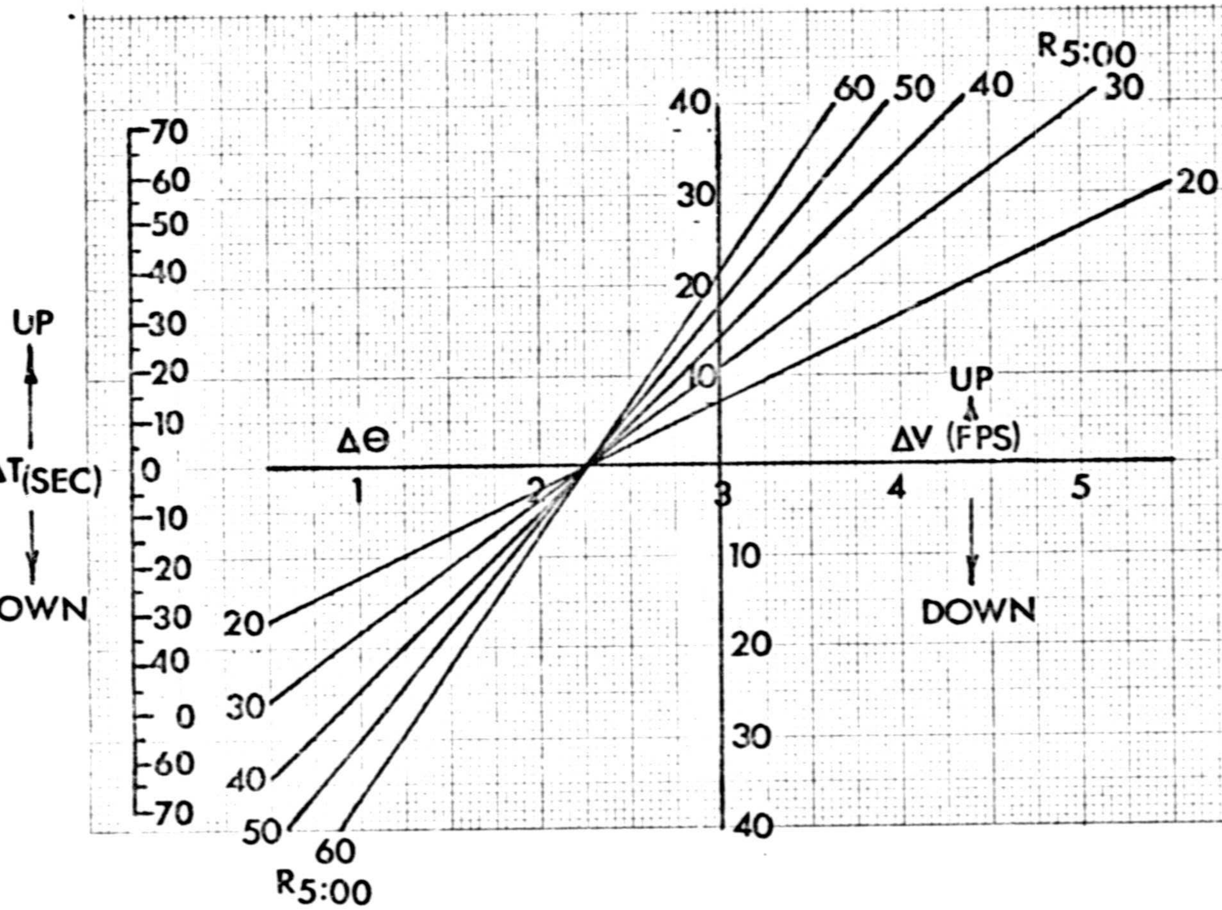
APPENDIX "A"

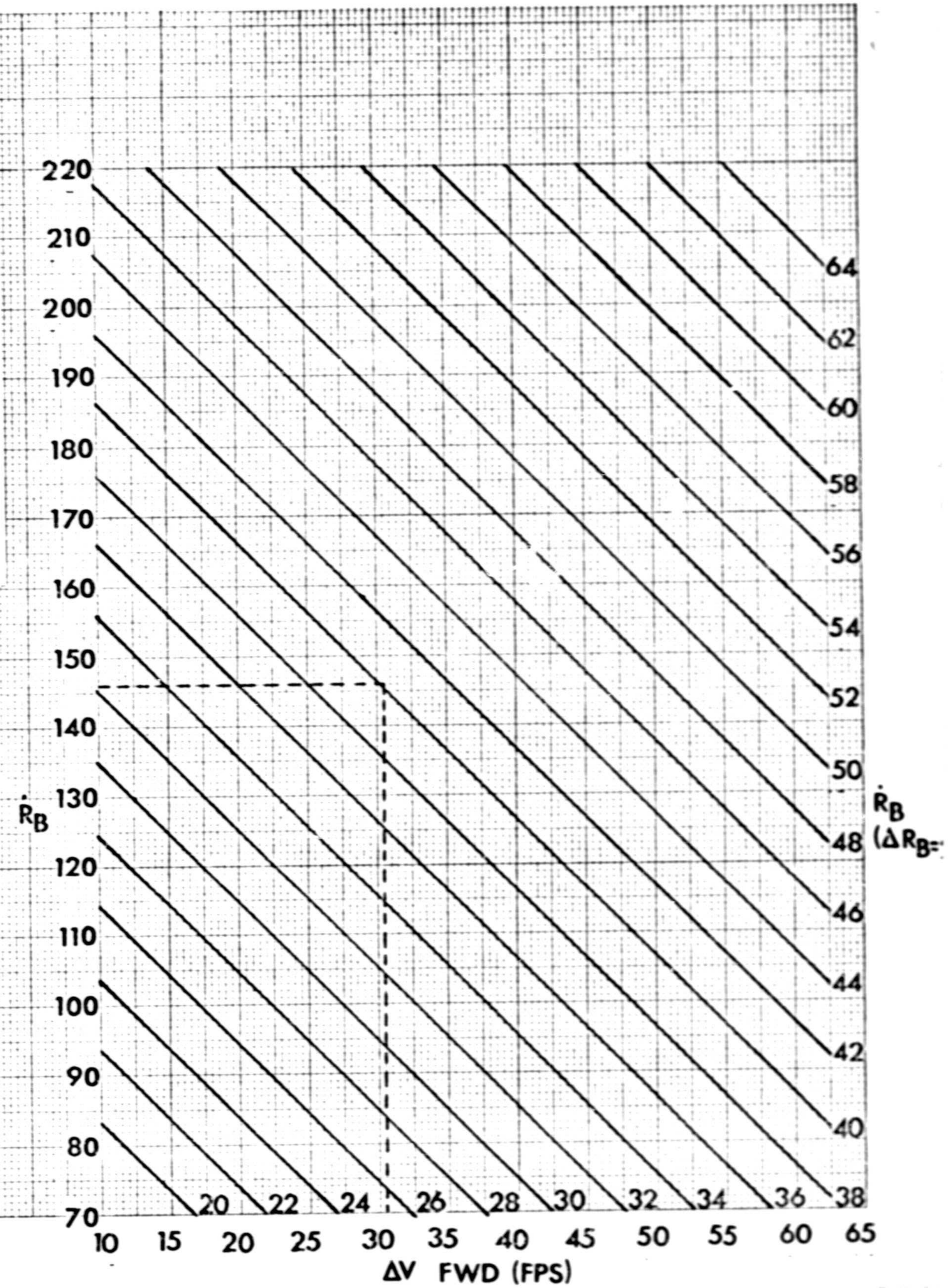
TERMINAL PHASE INITIATION

PT	TIME	e	e_{NOM}	R	R_{NOM}	$\dot{R}$	$\dot{R}_{NOM}$
A	-8:00	____	19.36	____	44.4	____	148.2
B	-5:00	____	-21.63	$\square$	40.1	$\square$	146.0
		$-\Delta\theta$ $\square$	2.27				

TPI 0:00

AXIS	DIR	ΔV	ΔT	USED
Z	FWD	____	____	____
X	U–D	____	____	____





FIRST CORRECTION

DT 8·16·67

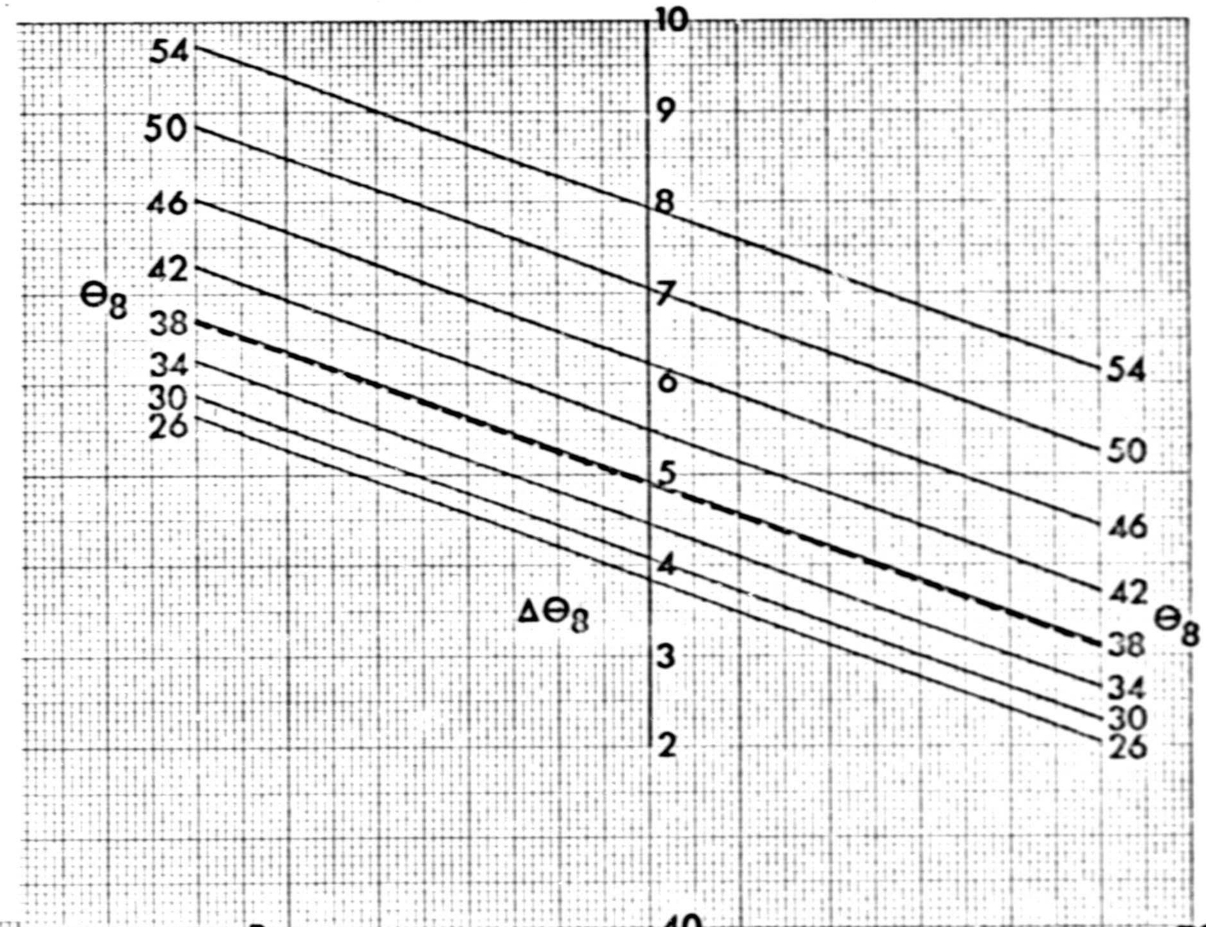
PT	TIME	Θ	Θ_{NOM}	R	R_{NOM}	$\dot{R}$	$\dot{R}_{NOM}$
A	5:00	____	32.91	____	25.7	____	159.5
B	8:00	____	37.82	$\boxed{}$	21.2	$\boxed{}$	147.5

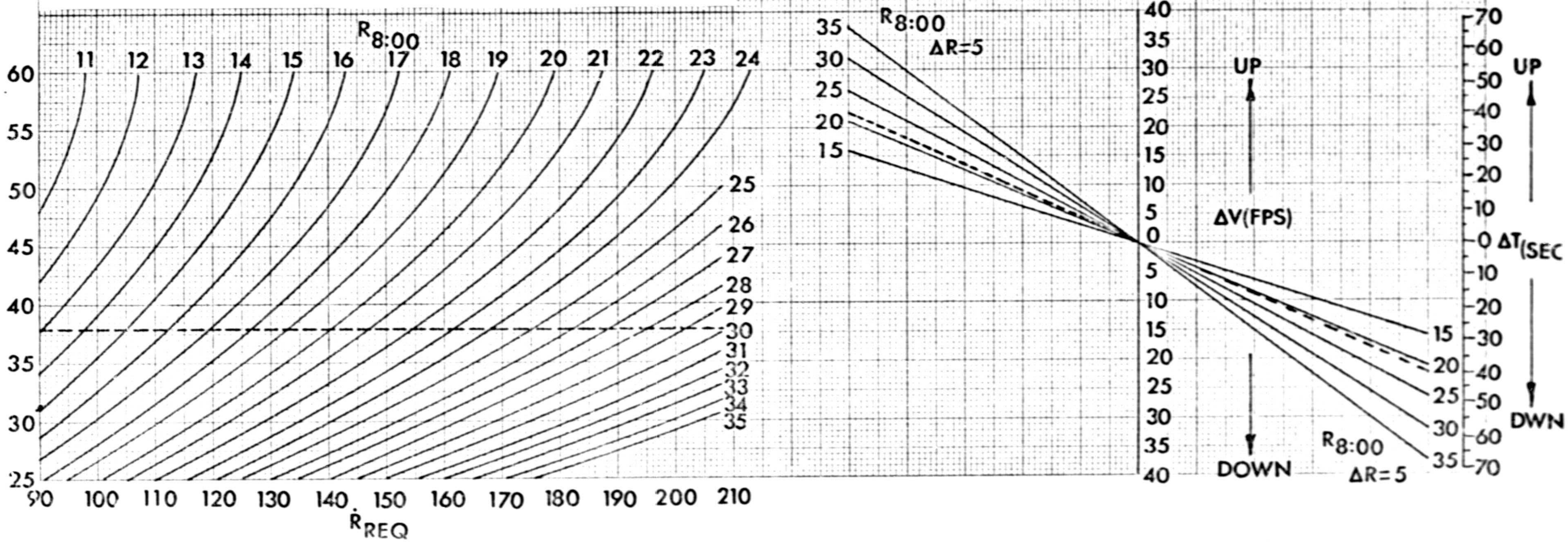
$\Delta\Theta$ $\boxed{}$ 4.91

$\dot{R}_{REQ}$ ____

$\dot{R}_{8:00}$ ____ $\boxed{}$

AXIS	DIR	ΔV	ΔT	USED
Z	F–A	____	____	____
X	U–D	____	____	____





SECOND CORRECTION

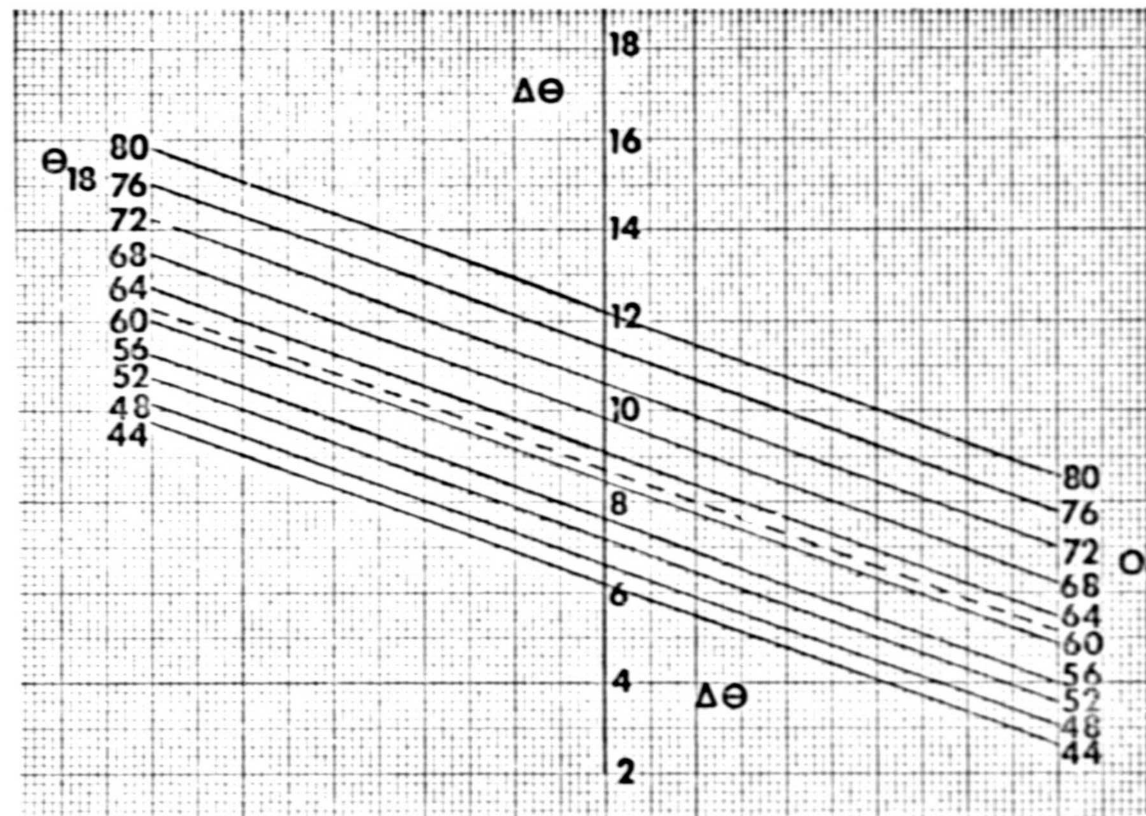
TIME	θ	θ_{NOM}	R	R_{NOM}	$\dot{R}$	$\dot{R}_{NOM}$
15:00		52.96		12.2		109.00
18:00		61.60		55200		90.04
$\Delta\theta$		8.64				

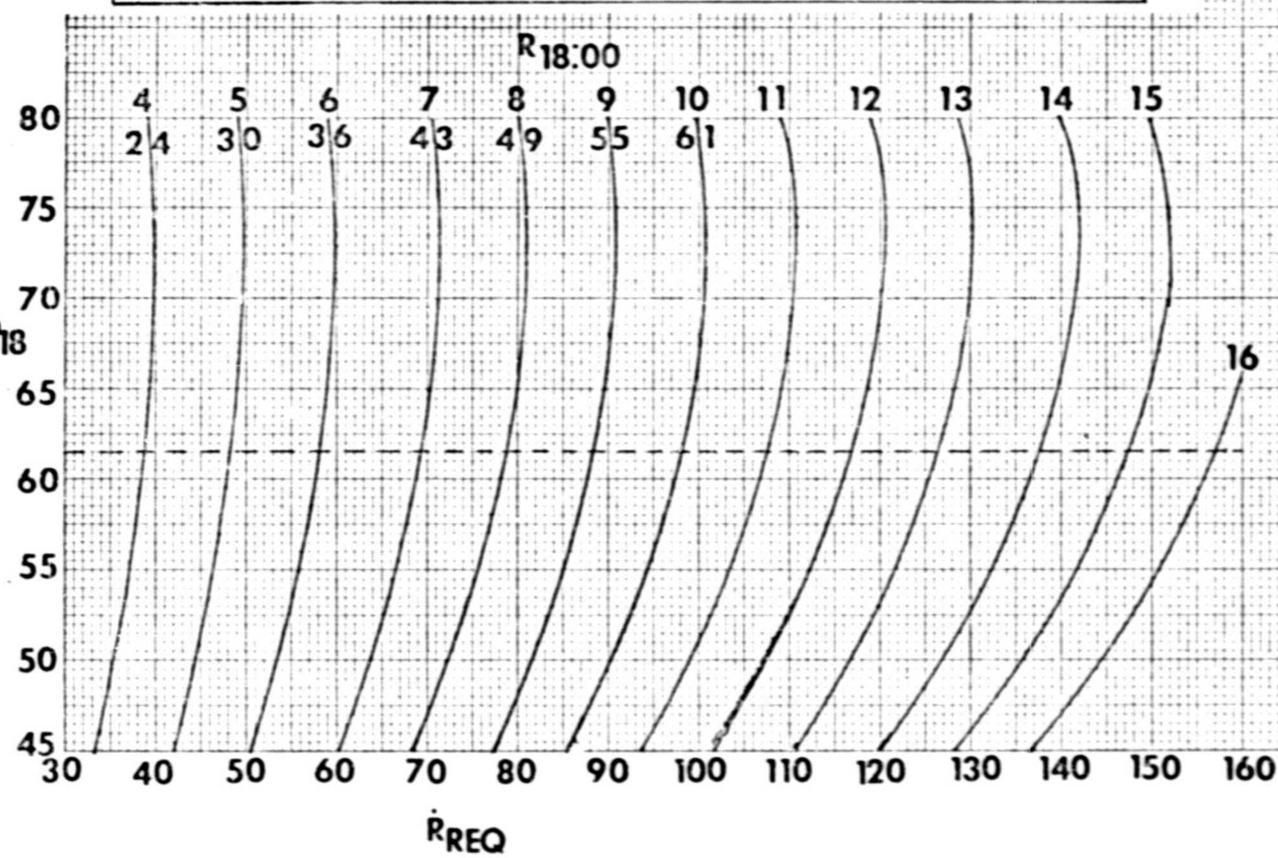
$\dot{R}_{REQ}$ ________

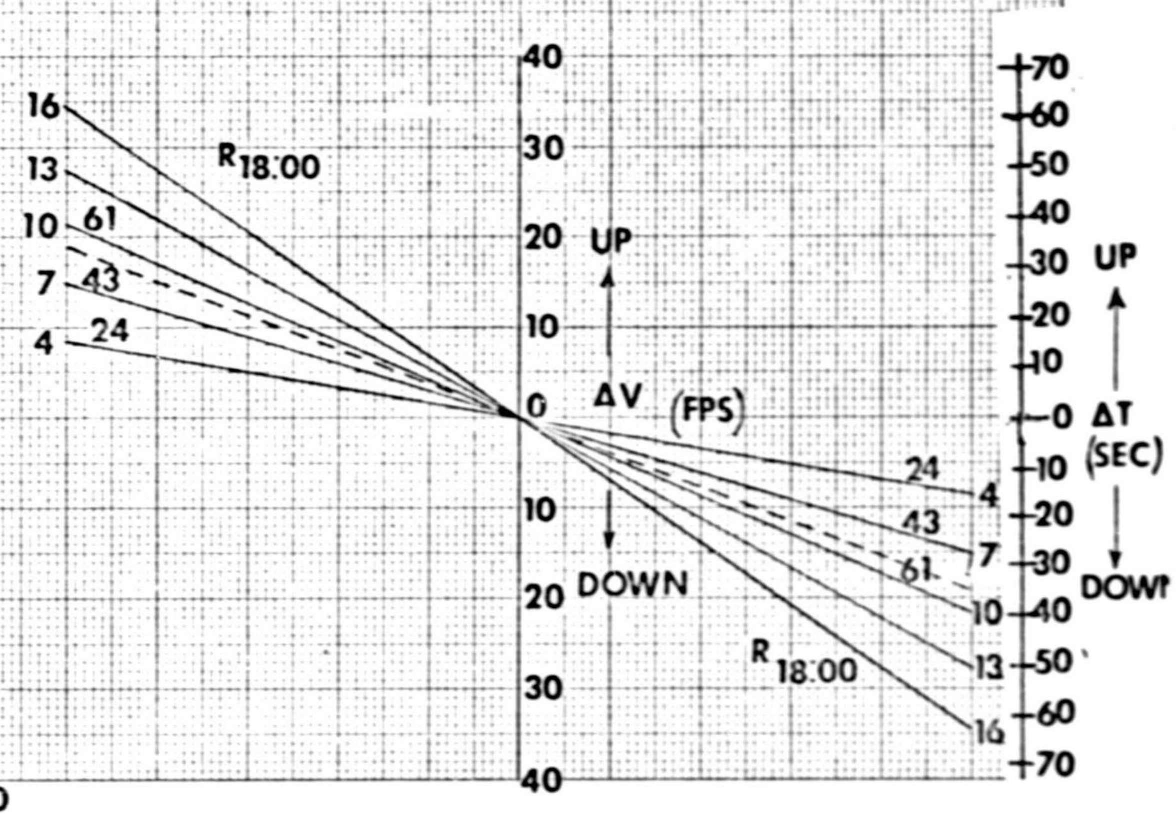
$\dot{R}_{18:00}$ = ________

ΔV_Z ________

AXIS	DIR	ΔV	ΔT	USED
Z	F–A			
X	U–D			







20	70	44	76	27	99	19	45
21	74	39	71	28	104	16	42
22	78	35	65	29	108	14	40
23	82	31	60	30	113	12	38
24	86	28	56	31	118	9	37
25	90	25	51	32	122	7	36
26	95	22	48	33	126	5	36